

Projections and best approximations

1. A velocity vector that gives the direction on a straight isolated road of a rescue vehicle is $\mathbf{v} = \begin{pmatrix} 13.2 \\ -51.5 \end{pmatrix}$. A drone has spotted a plane crash at a point $\mathbf{u} = \begin{pmatrix} 93.1 \\ -188.4 \end{pmatrix}$. How far should the rescue vehicle travel down the road before dropping off a crew to travel cross country to the location of the plane crash (assuming we want to minimize the distance the crew must travel cross country).
2. How many hours will the crew drive before reaching the drop-off position in Question 1?
3. Suppose you would like to approximate a sound using a single tone, as you only have a single oscillator, and therefore can only produce one specific frequency. In reality, such vectors would be much larger, but for now, assume the sound you would like to approximate $\mathbf{w}$ and the frequency you are able to reproduce $\mathbf{f}_1$ are given by

$$\mathbf{w} = \begin{pmatrix} 51 \\ 42 \\ 7 \\ -33 \\ -44 \\ -37 \\ -9 \\ 31 \end{pmatrix} \quad \text{and} \quad \mathbf{f}_1 = \begin{pmatrix} 1 \\ 0.707 \\ 0 \\ -0.707 \\ -1 \\ -0.707 \\ 0 \\ 0.707 \end{pmatrix},$$

respectively. What is the frequency closest to the target sound you can produce?

4. Suppose you can produce a second frequency given by

$$\mathbf{f}_2 = \begin{pmatrix} 0 \\ 0.707 \\ 1 \\ 0.707 \\ 0 \\ -0.707 \\ -1 \\ -0.707 \end{pmatrix}.$$

Is this frequency orthogonal to the first frequency vector $\mathbf{f}_1$?

5. As you may have found, the second frequency is orthogonal, and therefore, the best approximation of the target sound is the sum of the projections on to each of the two frequencies. What is this best approximation?